THE QUASISTATIONARY ONE-DIMENSIONAL MOTION OF AERATED OIL AND GAS

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The problem of the one-dimensional motion of aerated oil and gas arose in connection with the design of a development of the Samotlorsk oilfield, level A of which is a single massive stratum with a gas cap in the upper part and ground water in the underlying part. It is assumed that the area of the field is covered with a network of wells at a given spacing. The well filters are located arbitrarily in a single area which is called the plane of oil sampling. The position of the gas-oil contact and the water-oil contact are considered as functions of the time in the sampling plane in order to determine the depletion of the stock of oil and also to locate the sampling plane most advantageously in depth.

The problem falls into two parts: the motion of the gas–oil contact and the plane in which the oil is subject to saturation pressure by the gas; and the motion of the water–oil contact and the plane of saturation pressure. We take the sampling plane as the plane from which measurements are made. The x axis is taken as the axis of the change in the positions of the gas–oil contacts. Let L denote the distance from the sampling plane in the upper boundary of the gas cap: s_1 is the coordinate of the position of the plane of pressure saturation in the problem of the motion of the gas–oil contact; s_2 is the coordinate of the position of the gas–oil contact; s_3 is the coordinate of the position of the plane saturation in the problem of the motion of the water–oil contact; s_4 is the coordinate of the position of the water–oil contact.

We divide s_1, s_2, s_3 and s_4 by L and adopt the nondimensional coordinates u_1, u_2, u_3 and u_4; l_1 and l_2 are the nondimensional coordinates of the initial position of the gas–oil contact and the water–oil contact, respectively.

Suppose that at the initial moment of time the pressure in the stratum is $p^\circ > p_1$, where p_1 is the saturation pressure under which dissolved gas is separated out from the oil. We assume that in the process of developing the stratum the pressure in the sampling plane is fixed at a constant value and is equal to $p_0 < p_1$.

In addition we assume the following:

1) the generalized Darcy law holds for the oil pressure occupying the proportion σ of unit volume of the interstitial space [$f(\sigma)$ is the relative permeability of the oil]

$$v = -\frac{k}{\mu} f(\sigma) \frac{\partial p}{\partial x} \tag{1}$$

2) the oil density ρ is effectively constant; ρ = const (it is independent of the quantity of extracted gas);

3) gas extraction occurs instantaneously and depends on the pressure drop below the saturation pressure

$$\sigma = 1 - a(p_1 - p) \tag{2}$$

where a is a small quantity of dimension atm^{-1};

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4) the region occupied by the gas, $x \in (s_2, L)$, is remote from the region of strong pressure perturbations so that, assuming the process is isothermal, we can write

$$p_* (L - s_2) = p^\circ (L - l)$$

where $l = s_2(0)$ is the initial boundary between the oil and the gas, p_* is the variable pressure in the region of the gas as it expands;

5) in the zone $x \in [s_1 (t), s_2 (t)]$ the motion is rigid, so that

$$p = a_1 x + b_1$$

where a_1, b_1 are determined from the boundary conditions

$$x = s_1, \quad p = p_1, \qquad x = s_2, \quad p = p_* \tag{3}$$

From this

$$p = \frac{p^\circ (L - l) / (L - s_2) - p_1}{s_2 - s_1} (x - s_1) + p_1 \tag{4}$$

The kinematic condition for $x = s_2$ yields

$$m \frac{ds_2}{dt} = - \frac{k}{\mu} \frac{\partial p}{\partial x} = - \frac{k}{\mu} \frac{p^\circ (L - l) / (L - s_2) - p_1}{s_2 - s_1} \tag{5}$$

where m is the porosity of the stratum.

Equation (5) has to be integrated subject to the initial condition

$$s_2 (0) = l$$

6) in the zone $x \in (0, s_1)$ the oil moves with the gas.

By the Slikhter–Kozena equation, $k = Am^3/(1 - m)^2$, where A is a constant depending on the grain size of the skeleton of the porous medium. Replacing m by $m\sigma$, we have

$$k (\sigma) = A \frac{m^3 \sigma^3}{(1 - m\sigma)^2}$$

and for the phase permeability [1]

$$f (\sigma) = \frac{k (\sigma)}{k} = \frac{(1 - m)^2}{(1 - m\sigma)^2} \sigma^3 \tag{6}$$

The combination of the law of mass conservation for the oil and Darcy's law yields

$$\frac{k}{\mu} \frac{\partial}{\partial x} \left[\frac{(1 - m)^2}{(1 - m\sigma)^2} \sigma^3 \frac{\partial p}{\partial x} \right] = \frac{m \partial \sigma}{\partial t}$$

If we substitute (2) for σ in this equation and simplify, putting $[1 - a(p_1 - p)]^n \approx 1 - na (p_1 - p)$ $(n = 2, 3)$, we obtain

$$\frac{k}{\mu m a} \frac{\partial}{\partial x} \left\{ \left[1 - \frac{3 - m}{1 - m} a (p_1 - p) \right] \frac{\partial p}{\partial x} \right\} = \frac{\partial p}{\partial t} \tag{7}$$

Let q denote the pressure of the rock bed lying above the roof of the stratum. We can reduce Eq. (7) to nondimensional form by putting

$$\xi = \frac{p}{q}, \quad \xi_1 = \frac{p_1}{q}, \quad \xi_2 = aq, \quad y = \frac{x}{L}, \quad \tau = \frac{kt}{\mu m a L^2}$$

$$(\alpha \xi + \beta) \frac{\partial^2 \xi}{\partial y^2} + \alpha \left(\frac{\partial \xi}{\partial y} \right)^2 = \frac{\partial \xi}{\partial t}, \qquad \alpha = \frac{(3 - m) \xi_2}{1 - m}, \qquad \beta = 1 - \alpha \xi_1 \tag{8}$$

Equation (8) has to be integrated in the region $y \in (0, u_1)$, $\tau > 0$ under the conditions

$$\xi (0, \tau) = \xi_0, \quad \xi (u_1, \tau) = \xi_1, \quad u_1 (0) = 0 \tag{9}$$

$$\frac{\partial \xi}{\partial y} (u_1, \tau) = \frac{\xi^\circ (1 - l_1) / (1 - u_2) - \xi_1}{u_2 - u_1} \qquad \left(l_1 = \frac{l}{L}, \ \xi_0 = \frac{p_0}{q}, \ \xi^\circ = \frac{p^\circ}{q} \right)$$

We can put ξ in the form

$$\xi = c_0 + c_1 \frac{y}{u_1} + c_2 \frac{y^2}{u_1^2} \tag{10}$$

By using (9) in (4) we can find c_0, c_1, c_2:

$$c_0 = \xi_0, \quad c_1 = 2(\xi_1 - \xi_0) - [\xi^\circ (1 - l_1) / (1 - u_2) - \xi_1] u_1 / (u_2 - u_1)$$
$$c_2 = -c_1 + \xi_1 - \xi_0$$

We integrate (8) with respect to y from 0 to u_1. Noting (10), we obtain an equation containing $du_1/d\tau$ and $du_2/d\tau$. We combine this with equation (5), rewritten in nondimensional variables. We obtain the system of equations

$$\frac{du_1}{d\tau} = f_1(u_1, u_2), \quad \frac{du_2}{d\tau} = -f_2(u_1, u_2)$$
$$f_1(u_1, u_2) = \left\{\frac{B_0 (u_2 - u_1)^2}{u_1} + u_1^2 f_2(u_1, u_2)\left[\frac{\varepsilon_1 (1 + u_1 - 2u_2)}{(1 - u_2)^2} - \varepsilon_2\right]\right\}$$
$$\times \left[(\varepsilon_0 - \xi_1)(u_2 - u_1)^2 - u_1 (2u_2 - u_1)\left(\frac{\varepsilon_1}{1 - u_2} - \varepsilon_2\right)\right]^{-1} \tag{11}$$
$$f_2(u_1, u_2) = \xi_2 \frac{\xi^\circ (1 - l_1) / (1 - u_2) - \xi_1}{u_2 - u_1}$$

where

$$B_0 = 2c_2(\beta + c_0\alpha) + \alpha(3c_1c_2 + c_1^2 + 2c_2^2)$$
$$\varepsilon_0 = {}^2/_3\xi_1 + {}^1/_3\xi_0, \quad \varepsilon_1 = {}^1/_6\xi^\circ (1 - l_1), \quad \varepsilon_2 = {}^1/_6\xi_1$$

Equations (11) are to be integrated under the conditions

$$u_1(0) = 0, \qquad u_2(0) = l_1 \tag{12}$$

As $t \to \infty$ the functions u_1 and u_2 tend to a constant u_*, where $\xi_* = p_*/q \to \xi_1$. Then

$$u_* = 1 - \xi^0/\xi_1 (1 - l_1)$$

i.e., the gas begins to pass from the gas cap into the well after u_2 becomes equal to u_*.

We can consider two special cases:

for $l_1 = 1$ we have $u_* = 1$, i.e., there is no gas cap;

for $u_* = 0$, we have $l_1 = 1 - \xi_1/\xi^\circ$, i.e., for such a ratio of the quantities of oil and gas there is complete extraction of the oil;

as $\tau \to 0$

$$c_1 = 2\gamma, \quad c_2 = -\gamma \; (\gamma = \xi_1 - \xi_0)$$
$$u_1 \approx 2\sqrt{B\tau}, \quad u_2 \approx l_1 - \frac{\xi_2}{l_1}(\xi^\circ - \xi_1)\tau \quad \left(B = \frac{\gamma(\beta + \alpha\xi_0)}{\xi_1 - \varepsilon_0}\right)$$

From these equations we have computed the initial values of the oil for $\tau = 1 \cdot 10^{-10}$ given the initial conditions

$$u_1 = 34 \cdot 10^{-6}, \quad u_2 = l_1$$

In the zone $y \in (0, u_3)$ the oil pressure is accompanied by extraction of dissolved gas. Here u_3 is the boundary on which the pressure is ξ_1.

For $y \in (u_3, u_4)$ the motion of the oil is rigid so that

$$\xi = a_2 y + b_2 \tag{13}$$

Here u_4 is the boundary on which the ground water pressure is maintained at a constant value equal to ξ°.

In this case we have the boundary conditions

$$y = u_3, \quad \xi = \xi_1; \qquad y = u_4, \quad \xi = \xi^\circ \tag{14}$$

which, in the domain $y \in (u_3, u_4)$ yields

$$\xi = \frac{\xi^\circ - \xi_1}{u_4 - u_3} y + \frac{\xi_1 u_4 - u_3 \xi^\circ}{u_4 - u_3}$$

The kinematic condition for $y = u_4$, analogous to (5), yields

$$\frac{du_4}{d\tau} = -\frac{\xi_2 (\xi^\circ - \xi_1)}{u_4 - u_3} \tag{15}$$

In $y \in (0, u_3)$ we have the boundary conditions and the equation

$$\xi(0, \tau) = \xi_0, \quad \xi(u_3, \tau) = \xi_1, \quad \frac{\partial \xi}{\partial y} = \frac{\xi^\circ - \xi_1}{u_4 - u_3} \tag{16}$$

Putting

$$\xi = \omega_0 + \omega_1 \frac{y}{u_3} + \omega_2 \frac{y^2}{u_3^2}$$

we obtain, as above,

$$\frac{du_3}{d\tau} = \left\{D \frac{(u_4 - u_3)^2}{u_3} - \varepsilon_3 u_3^2 \frac{du_4}{d\tau}\right\} \Big/ [(\varepsilon_0 - \xi_1)(u_4 - u_3)^2 - \varepsilon_3 u_3 (2u_4 - u_3)] \tag{17}$$

where

$$D = 2\omega_2 (\beta + \alpha\omega_0) + \alpha (3\omega_1\omega_2 + \omega_1^2 + 2\omega_2^2)$$

$$\alpha = (3 - m)\xi_2 / (1 - m), \quad \beta = 1 - \alpha \xi_1, \quad \varepsilon_3 = {}^1/_6 (\xi^\circ - \xi_1)$$

$$\omega_0 = \xi_0, \quad \omega_1 = 2(\xi_1 - \xi_0) - (\xi^\circ - \xi_1) u_3 / (u_4 - u_3),$$

$$\omega_2 = -\omega_1 + \xi_1 - \xi_0$$

We combine Eqs. (15) and (17) and seek the solution subject to the initial conditions

$$u_3(0) = 0, \qquad u_4(0) = l_2 \tag{18}$$

As $\tau \to 0$, $\omega_1 \to 2\gamma$, $\omega_2 \to -\gamma$, and so from (17) we have

$$u_3 \approx 2\left[\frac{\gamma(\beta + \alpha\xi_0)\tau}{\xi_1 - \varepsilon_0}\right]^{1/2}, \qquad u_4 \approx l_2$$

By solving the above differential equations for various values of l_1 and l_2, we can choose these parameters so that the water–oil contact reaches the sampling plane more rapidly than the gas–oil contact.

Having chosen the optimum variant of the position of the sampling plane from the surface of the ground, we seek to determine the depletion of the resources of the oil bed after a given interval of time.

We assume that $u_1(\tau)$, $u_2(\tau)$, $u_3(\tau)$, $u_4(\tau)$ are known functions, after solving the appropriate system of differential equations.

The flow of oil in unit time through unit area of the rock vertically is found from Darcy's equation

$$Q = \frac{k}{\mu} \frac{\partial p}{\partial x} f(\sigma) \quad \text{for} \quad x = 0 \tag{19}$$

We can write (19) in nondimensional variables:

$$Q_1 = \frac{\partial \xi}{\partial y} = \frac{Q\mu L}{kqf(\sigma_0)}, \quad \xi = \frac{p}{q},$$

$$y = \frac{x}{L}, \quad f(\sigma_0) = f(\sigma)|_{x=0}$$

The total flow Q_2 of oil through unit area in the time from 0 to τ can be expressed by the sum of the integrals

$$Q_2 = \int_0^\tau \frac{c_1}{u_1} d\tau + \int_0^\tau \frac{\omega_1}{u_3} d\tau$$

The calculations are carried out for the values

$$p_0 = 170 \text{ atm}, p_1 = 171 \text{ atm}, \quad p^\circ = 172 \text{ atm}, q = 350 \text{ atm}, L = 154 \text{ m}$$
$$\mu = 1.97 \text{ cp}, \quad a = 9.76 \cdot 10^{-4} \text{ atm}^{-1}, m = 0.22, \ k = 0.22 \text{ darcy}$$

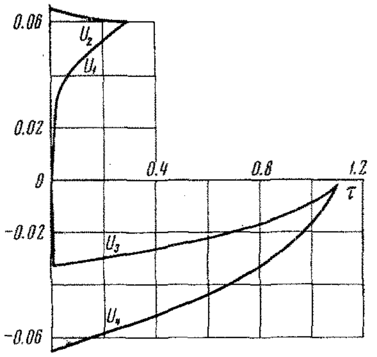

Fig. 1

The coefficient *a* was found, for the chosen parameter, under the guidance of [2] for a volume coefficient of the oil bed of 1.2 and a gas compressibility coefficient of 0.85. The chosen parameters were close to the parameters of the Samotlorsk oil field. We quote the results for the optimal variant of the position of the sampling plane.

l_1	L(m)	τ_1	l_2	L(m)	τ_1
0.09434	159	0.42	0.031446	159	0.253
0.0649	154	0.30	0.0649	154	1.1
0.03356	149	0.17	0.1007	149	2.66

Here τ_1 is the time at which the water–oil or the gas–oil contact reaches the moving boundary on which the pressure is ξ_1. Graphs of $u_1(\tau)$, $u_2(\tau)$, $u_3(\tau)$, $u_4(\tau)$ are shown in Fig. 1 for $l_1 = 0.0649$, $l_2 = 0.0649$. The calculations were made using a digital computer.

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